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Editorial Board

Title

CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH SIX SIGMA QUALITY LEVELS WITH CHAIN SAMPLING PLAN-(0,1) AS ATTRIBUTE PLAN

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Abstract:

Six Sigma is a concept, a process, a measurement, a tool, a quality philosophy, a culture and a management strategy for the improvement in the system of an organization, in order to reduce wastages and increase the profit to the management and enhance satisfaction to the customers. Motorola (1980) first adopted the concept of six sigma in their organization and established that it can produce less than 3.4 defects per million opportunities. Focusing on reduction of defects will result in more profit to the producer and enhanced satisfaction for the consumer. The concept of Six Sigma can be applied in the process of quality control in general and Acceptance sampling in particular.

In this paper a procedure for the construction and selection of Mixed Sampling Plan indexed through Six Sigma Quality level having the Chain Sampling Plan-(0,1) Plan [ChSP-(0, 1)]) as attribute plan is presented. The plans are constructed using SSQL-1 and SSQL-2 as indexing parameters. Tables are constructed for easy selection of the plan.

Keywords: Six Sigma Quality Level, Poisson Distribution, Operating Characteristic Curve, Mixed Sampling Plan, ChSP-(0,1) Plan.

Introduction:

Mixed sampling plan is a two stage sampling procedure involving variables inspection in the first stage and attributes inspection in the second stage if the variables inspection of the first sample does not lead to acceptance. Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed sampling plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary.

It is the usual practice that while selecting a sampling inspection plan, to fix the operating characteristic (OC) curve in accordance with the desired degree of discrimination. The sampling plan is in turn fixed by suitably chosen parameters. The entry parameters used in the acceptance sampling literature are acceptable quality level (AQL), limiting quality level (LQL), indifference quality level (IQL) and maximum allowable percent defective (MAPD) and maximum allowable average outgoing quality(MAAOQ). Several authors have provided procedures to design the sampling plans indexed by these parameters.

The mixed sampling has been designed under two cases of significant interest. In the first case the sample size n_1 is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given.

The mixed sampling plans are initially introduced by Dodge (1932) and later developed by Bowker and Goode (1952). Schilling (1967) has given a method for determining the operating characteristics for mixed variables-attributes sampling plans. Dodge and Stephens (1966) developed the Chain sampling plan of the type ChSP-(0, 1) which is generalization of Dodge's (1955) chain sampling plan of the type ChSP-1. These plans can be used for both small and large samples, but is particularly useful when samples necessarily be small (especially tests are costly or destructive) and a more complete discussion of chain sampling plan is provided by Schilling (1982). Radhakrishnan and Sampathkumar (2009) constructed mixed sampling plans with ChSP-(0,1) plan as attribute plan for various entry parameters. Further Radhakrishnan (2009) constructed Six Sigma based Single Sampling Plans Indexed through Six Sigma Quality Levels with Poisson distribution, Weighted Poisson distribution and IRPD as the base line distributions. Radhakrishnan and Saravanan (2009, 2010) constructed mixed dependent sampling plan with single sampling and chain sampling plan as attribute plan. Radhakrishnan and Balamurugan (2011) constructed control charts based on six sigma initiatives for Xbar using Standard deviation. Radhakrishnan and glorypersial (2011) constructed mixed sampling plans indexed through six sigma quality levels with Double Sampling Plan as Attribute Plan. Radhakrishnan and Glorypersial (2011) constructed mixed sampling plan indexed through six sigma quality levels with Conditional Double Sampling Plan as Attribute Plan.

This paper deals with the construction of mixed variables – attributes sampling plan (independent case) using ChSP-(0,1) Plan as attribute plan indexed through Six Sigma Quality levels. Tables are constructed for easy selection of the plan and illustrations are also provided.

Glossary of Symbols:

The symbols used in this paper are as follows:

p : submitted quality of lot or process

$P_a(p)$: probability of acceptance for given quality p

n_1: sample size for variable sampling plan

$n_{1,2}$: first sample size for attribute sampling plan

$n_{2,2}$: second sample size for attribute sampling plan

β_j : probability of acceptance for lot quality p_j

β_j': probability of acceptance assigned to first stage for percent defective p_j

β_j'' :probability of acceptance assigned to second stage for percent defective pj

k: variable factor such that a lot is accepted if $\bar{X} \leq A = U - k\sigma$

Operating Procedure of Mixed Sampling Plan with ChSP-(0,1) Plan as attribute plan:

- Determine the four parameters of the mixed plan n_1, n_2, k, k_1 and k_2 with reference to ASN and OC curves
- Take a random sample of size n_1 from the lot assumed to be large
- If the sample average $\bar{X} \leq A = U - k\sigma$, accept the lot
- If the sample average $\bar{X} > A = U - k\sigma$, take a second sample of size n_2
- Test the second sample against a given attributes criterion

  (i) The number of nonconformities in each sample is recorded, as well as cumulative number of non-conformities found so far.

  (ii) Accept the lot associated with each new sample as long as no non-conformity is found.

  (iii) Once k_1 lots have been accepted, accept subsequent lots as long as the cumulative number of nonconformities is no greater than one.

  (iv) Once $k_2 > k_1$ lots have been accepted, cumulate the number of non-conformities only over the most recent k_2 lots, and continue to accept lots as long as this cumulative number of non-conformities is no greater than one.

(v) If, at any stage, the cumulative number of nonconformities becomes greater than one, reject the current lot and return to step (i).

Conditions for applications:

(i) Production process should be steady and continuous

(ii) Lots are submitted sequentially in the order of their production

(iii) Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective

(iv) Human involvement should be less in the manufacturing process

Definition of SSQL-1 and SSQL-2:

The proportion defective corresponding to the probability of acceptance of the lot as 1-3.4 x 10^{-6}, (the concept of six sigma quality suggested by Motorola (1980)) in the OC curve is termed as Six Sigma Quality Level-1 (SSQL-1). This new sampling plan is constructed with a point on the OC curve (SSQL-1, $1-\alpha_1$), where $\alpha_1=3.4 \times 10^{-6}$ suggested by Radhakrishnan and Sivakumaran (2008) and Radhakrishnan(2009). Further the proportion defective corresponding to the probability $2\alpha_1$ in the OC curve is termed as Six Sigma Quality Level-2 (SSQL-2). This new sampling plan is constructed with a point on the OC curve (SSQL-2, β_1), where $\beta_1=2\alpha_1$ suggested by Radhakrishnan and Sivakumaran (2008) and Radhakrishnan(2009).

Designing the Mixed Sampling Plan when a single point on the OC curve is known:

The procedure for the construction of mixed variables – attributes sampling plans is provided by Schilling (1967) for a given 'n_1' and a point 'p_1' on the OC curve. A modified procedure for the construction of independent mixed variables – attributes sampling plan for a given SSQL-1, SSQL-2 and 'n_1' is given below.

- Split the probability of acceptance (β_j) as determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as

  $A = U - k\sigma = U - [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma$, where z (t) is the standard normal variate corresponding to 't' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$
- Determine the sample average $\bar{X}$. If a sample average $\bar{X} > A = U - k\sigma$, take a second stage sample of size 'n_2' using an attribute sampling plan.
- Now determine β_1'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_1'' = (\beta_1 - \beta_1') / (1-\beta_1')$.
- Determine the appropriate second stage sample of size 'n_2' and 'c' from $P_a(p) = \beta_1''$ for p=SSQL-1.
- Determine β_2'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_2'' = (\beta_2 - \beta_2') / (1-\beta_2')$.
- Determine the appropriate second stage sample of size 'n_2' and 'c' from $P_a(p) = \beta_2''$ for p=SSQL-2.

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with any attribute plan indexed through SSQL-1 and SSQL-2.

Operating Characteristic function:

Under the assumption of Poisson model , the OC function of the independent mixed sampling plan having ChSP-(0,1) Plan is given by

$$P_a(p)=P(\bar{X} \le A) + [1 - P(\bar{X} \le A)]\left[\frac{P_0(1-P_0) + P_1P_0^{k_1}(1-P_0^{k_2-k_1})}{1-P_0+P_1P_0^{k_1}(1-P_0^{k_2-k_1-1})}\right],\ k_2 > k_1$$

Where P_0 = Probability for getting exactly zero nonconformities in a sample of size n_2

P_1 = Probability of getting exactly one nonconformity in a sample of size n_2

k_1 = minimum number of successive samples required to be free from nonconformities before cumulation can take place

k_2 = maximum number of successive samples over which the cumulation of nonconformities take place.

Construction of MSP with ChSP-(0,1) Plan as attribute plan indexed through SSQL-1:

In this section the mixed sampling plan indexed through SSQL-1 is constructed. A point on the OC curve can be fixed such that the probability of acceptance of fraction defective SSQL-1 is β_1. The general procedure given by Schilling (1967) is used for constructing the mixed sampling plan as attribute plan indexed through SSQL-1 [for $\beta_1'' = (\beta_1 - \beta_1') / (1-\beta_1')$]. For β_1=0.9999966 and β_1'=0.50, the n_2SSQL-1 values are calculated for different values of k_1 and k_2 using visual basic program and is presented in Table1.

The sigma level of the process is calculated using the Process Sigma Calculator by providing the sample size and acceptance number.

Selection of the plan

Table 1 is used to construct the plans when SSQL-1, k_1 and k_2 are given. For any given values of SSQL-1, k_1 and k_2 one can determine n_2 value using $n_2 = n_2$SSQL-1/SSQL-1.

Example 1: Given SSQL-1=0.000001, k_1=2, k_2=4, and β_1'=0.50, the value of SSQL-1 is selected from Table 1 as 0.001400 and the corresponding sample size n_2 is computed as n_2= n_2SSQL-1/SSQL-1=0.001400/0.000001=1400, which is associated with 4.7 sigma level. For a fixed β_1'=0.50, the Mixed Sampling Plan with ChSP-(0,1) Plan as attribute plan are n_2=1400, k_1=2 and k_2=4 for a specified SSQL-1=0.000001.

Practical Application:

Suppose the plan with n_1 = 10, k = 1.5, k_1=2 and k_2=4 is to be applied to the lot-by-lot acceptance inspection of ball bearings for wheel hubs of a bicycle. The characteristic to be inspected is the "bearing diameters in mm" for which there is a specified upper limit (U) of 5.2 mm with a known standard deviation (σ) of 0.002 mm. In this example, U=5.2 mm, σ = 0.002 mm and k = 1.5

Now, in applying the variable inspection first, take a random sample of size n_1=10 from the lot. Record the sample results and find $\overline{X}$. If $\overline{X} \leq A = U - k\sigma$ =5.197 mm, accept the lot otherwise take a random sample of size n_2 =1400 and apply attribute inspection.

Under attribute inspection, the ChSP-(0,1) plan as attribute plan, if the manufacturer fixes the values β_1'=0.50, SSQL-1=0.000001 (1 non-conformity bearings out of 10 lakh) then select a sample of 1400 bearings from the same lot and count the number of defectives as well as the cumulative number of defective bearings. . If the measure of diameter of any bearing is greater than 5.197 mm, then it is termed as defective. Accept the lot of bearings no defective bearing is found. Once 2 lots have been accepted, accept subsequent lots of bearings as long as the cumulative number of defective bearing is not greater than one. Once 4 lots of bearing have been accepted, cumulate the number of defective bearings only over the most recent 4 lots, and continue to accept the lots of bearings as long as this cumulative number of defective bearing is not greater than one. Then at any stage the cumulative number of defective bearing becomes greater than one, reject the current lot of bearings packed and continue the process for the next coming lots and inform the management for improving the quality. The OC curve of the plan in Example 1 is presented in the Figure 1.

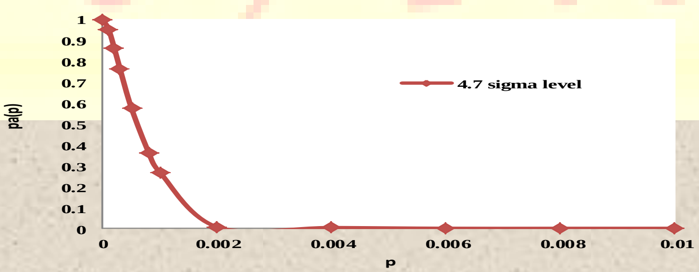

Figure 1. OC curve for the plan n_2=1400, k_1=2 and k_2=4

Construction of MSP with ChSP-(0,1) plan as attribute plan indexed through SSQL-2:

In this section the mixed sampling plan indexed through SSQL-2 is constructed. A point on the OC curve can be fixed such that the probability of acceptance of fraction defective SSQL-2 is β_2. The general procedure given by Schilling (1967) is used for constructing the mixed sampling plan as attribute plan indexed through SSQL-2 [for $\beta_2'' = (\beta_2 - \beta_2') / (1-\beta_2')$]. For β_2 = 0.0000068 and β_2' = 0.0000034, the n_2SSQL-2 values are calculated for different values of k_1 and k_2 using visual basic program and is presented in Table2.

Example 2: Given SSQL-2=0.02, k_1=1, k_2=3, and β_1'=0.50, the value of SSQL-2 is selected from Table 1 as 12.59178 and the corresponding sample size n_2 is computed as n_2= n_2SSQL-2/SSQL-2=12.59178 /0.02=630, which is associated with 4.5 sigma level. For a fixed β_1'=0.50, the Mixed Sampling Plan with ChSP-(0,1) Plan as attribute plan are n_2=630, k_1=1 and k_2=3 for a specified SSQL-2=0.02.

Practical Application:

Suppose the plan with n_1 = 10, k = 1.5, k_1=1 and k_2=3 is to be applied to the lot-by-lot acceptance inspection of aluminum foil of an air conditioner. The characteristic to be inspected is the "inner diameter of coil in mm" for which there is a specified upper limit (U) of 75 mm with a known standard deviation (σ) of 0.002 mm. In this example, U=75 mm, σ = 0.002 mm and k = 1.5

Now, in applying the variable inspection first, take a random sample of size n_1=10 from the lot. Record the sample results and find $\overline{X}$. If $\overline{X} \leq A = U - k\sigma$ =74.97 mm, accept the lot otherwise take a random sample of size n_2 =630 and apply attribute inspection.

Under attribute inspection, the ChSP-(0,1) plan as attribute plan, if the distributor fixes the values β_1'=0.50, SSQL-2=0.02 (2 non-conformity aluminum foils out of 100) then select a sample of 630 aluminum foils from the same lot and count the number of defectives as well as the cumulative number of defective aluminum foils. If the measure of diameter of any aluminum foils is greater than 74.97 mm, then it is termed as defective. Accept the lot of aluminum foils no defective aluminum foils is found. Once 1 lot has been accepted, accept subsequent lots of aluminum foils as long as the cumulative number of defective aluminum foil is not greater than one. Once 3 lots of aluminum foils have been accepted, cumulate the number of defective aluminum foils only over the most recent 3 lots, and continue to accept the lots of aluminum foils as long as this cumulative number of defective aluminum foil is not greater than one. Then at any stage the cumulative number of defective aluminum foils becomes greater than one, reject the current lot of aluminum foils packed and continue the process for the next coming lots and inform the management for improving the quality. The OC curve of the plan in Example 2 is presented in the Figure 2.

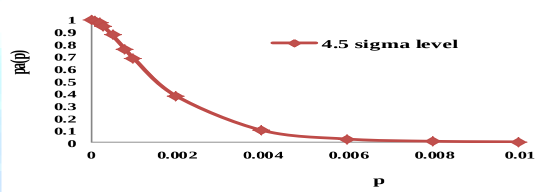

Figure 2. OC curve for the plan n_2=630, k_1=1 and k_2=3

Conclusion:

This paper provides a procedure to engineers for the selection of Mixed Sampling Plan through Six Sigma Quality Levels having ChSP-(0,1) plan as attribute plan. These plans are very effective in place of classical plans indexed through SSQL-1 and SSQL-2 and these plans are useful for the companies in developed and developing countries using six sigma quality initiatives in the manufacturing process. The procedure outlined in this paper can be used for other plans also.

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Table 1: Various characteristics of the MSP when (SSQL-1, β_1) is known with β_1 = 0.9999966, β_1' = .50.

k_1	k_2	n_2SSQL-1
1	2	0.002140
1	3	0.001660
1	4	0.001400
1	5	0.001240
1	6	0.001120
1	7	0.001030
1	8	0.000960
1	9	0.000900
1	10	0.000860
2	3	0.001660
2	4	0.001400
2	5	0.001240
2	6	0.001120
2	7	0.001030
2	8	0.000960
2	9	0.000900
2	10	0.000860
3	4	0.001400
3	5	0.001240
3	6	0.001120
3	7	0.001030
3	8	0.000960
3	9	0.000900

3	10	0.000860
4	5	0.001240
4	6	0.001120
4	7	0.001030
4	8	0.000960
4	9	0.000900
4	10	0.000850
5	6	0.001120
5	7	0.001030
5	8	0.000960
5	9	0.000900
5	10	0.000850
6	7	0.001030
6	8	0.000960
6	9	0.000900
6	10	0.000850
7	8	0.000960
7	9	0.000900
7	10	0.000850
8	9	0.000900
8	10	0.000850
9	10	0.000850

Table2: Various characteristics of the MSP when (SSQL-2 , β_2) is known with

β_2 = 0.0000068 and β_2' = 0.0000034.

k_1	k_2	n_2SSQL-2
1	2	12.59178
1	3	12.59178
1	4	12.59178
1	5	12.59178
1	6	12.59178
1	7	12.59178
1	8	12.59178
1	9	12.59178
1	10	12.59178
2	3	12.59174
2	4	12.59174
2	5	12.59174
2	6	12.59174
2	7	12.59174
2	8	12.59174
2	9	12.59174
2	10	12.59174
3	4	12.59174
3	5	12.59174
3	6	12.59174
3	7	12.59174
3	8	12.59174

3	9	12.59174
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5	6	12.59174
5	7	12.59174
5	8	12.59174
5	9	12.59174
5	10	12.59174
6	7	12.59174
6	8	12.59174
6	9	12.59174
6	10	12.59174